Arithmetic Sequences
Going Deeper

**Essential question:** Why is a sequence a function and how can you write a rule for an arithmetic sequence?

Give examples of functions for which the domain is restricted, in particular where the domain is the set of integers, and have students evaluate functions for given values of the domain.

### Standards for Mathematical Content

- **F-IF.1.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F-IF.1.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- **F-BF.1.1** Write a function that describes a relationship between two quantities.*
- **F-BF.1.1a** Determine an explicit expression, a recursive process … from a context.*
- **F-BF.1.2** Write arithmetic ... sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*
- **F-LE.1.2** Construct linear ... functions, including arithmetic ... sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*

### Vocabulary

- **sequence**
- **recursion**
- **rule**
- **arithmetic sequence**
- **common difference**

### Math Background

Students are familiar with the concept of functions and know that a function assigns exactly one output to each input. Students understand that a function has a domain and a range, and they have previously evaluated functions for inputs in their domains. Students have also written functions for real-world situations, and understand the use of function notation to both interpret and express statements in terms of a context.

### TEACH

#### 1. Engage

**Questioning Strategies**
- Can a sequence with repeating terms, such as 1, 1, 2, 3, 3, …, be a function? Yes, the domain of the function is the set of position numbers, and the set of terms is the range. Values can be repeated in the range of a function.
- Predict the next term in the sequence 1, 1, 3, 1, 5, 8, 4, 8, 2, 8, …, and describe the pattern.
  - Each term is $\frac{3}{8}$ more than the previous term, so the next term is $\frac{1}{8}$ more than $\frac{5}{8}$.

**Teaching Strategy**

Reinforce the concept that a sequence is a function by emphasizing the domain and range of a sequence. Write the positive integers 1, 2, 3, and so on across the top of the board in front of the class. Explain that, for a sequence, the positive integers correspond to the positions of the terms in the sequence. Then, have students write the terms of the sequence 1, 3, 5, 7, 9, … directly below the position numbers and draw arrows between each position number and its corresponding term. Identify the position numbers as the domain and the terms as the range; then, ask students to use what they know about relations to explain why this particular relation is a function.

Point out that position numbers can be consecutive integers that start at some number other than 1. A common alternative starting position number is 0. Regardless of what starting position number is used, a sequence is still a function. Students, however, must be careful when using function notation to identify the terms of a sequence. For the sequence 2, 4, 6, 8, …, for instance, $f(3) = 6$ when the starting position number is 1, but $f(3) = 8$ when the starting position number is 0.
Arithmetic Sequences

Going Deeper

Essential question: Why is a sequence a function and how can you write a rule for an arithmetic sequence?

1. **Engage**

Understanding Sequences

A **sequence** is an ordered list of numbers or other items. Each element in a sequence is called a **term**. For instance, in the sequence 1, 3, 5, 7, 9, . . . , the second term is 3.

Each term in a sequence can be paired with a position number, and these pairings constitute a function whose domain is the set of position numbers and whose range is the set of terms, as illustrated below. The position numbers are consecutive integers that typically start at either 1 or 0.

<table>
<thead>
<tr>
<th>Position number</th>
<th>nth term of sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>f(1) = 1</td>
</tr>
<tr>
<td>2</td>
<td>f(2) = 3</td>
</tr>
<tr>
<td>3</td>
<td>f(3) = 5</td>
</tr>
<tr>
<td>4</td>
<td>f(4) = 7</td>
</tr>
</tbody>
</table>

For the sequence shown in the table, you can write \( f(n) = 2n - 1 \), which can be interpreted as "the fourth term of the sequence is 7."

**Reflect**

1a. The domain of the function \( f \) defining the sequence 2, 5, 8, 11, 14, . . . is the set of consecutive integers starting with 0. What is \( f(0) \)? Explain how you determined your answer.

1b. How does your answer to Question 1a change if the domain of the function is the set of consecutive integers starting with 1?

1c. Predict the next term in the sequence 2, 5, 8, 11, 14, . . . . Explain your reasoning.

1d. Why is the relationship between the position numbers and the terms of a sequence a function?

1e. Give an example of a sequence from your everyday life. Explain why your example represents a sequence.

The numbers are a sequence because their order matters.

**Example**

Using a Recursive Rule to Generate a Sequence

Write the first 4 terms of the sequence \( f(n) = n^2 \). Assume that the domain of the function is the set of consecutive integers starting with 1.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n^2 )</th>
<th>( f(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>17</td>
</tr>
</tbody>
</table>

The first 4 terms are 1, 2, 5, and 17.

**Reflect**

2a. How could you use a graphing calculator to check your answer?

Sample answer: Enter the equation \( y = x^2 \) and make a table.

2b. Explain how to find the 20th term of the sequence.

Evaluate \( n^2 + 1 \) for \( n = 20; f(20) = 401 \). A recursive rule for a sequence defines the nth term by relating it to one or more previous terms.

**Example**

Using a Recursive Rule to Generate a Sequence

Write the first 4 terms of the sequence with \( f(1) = 3 \) and \( f(n) = f(n - 1) + 2 \) for \( n \geq 2 \). Assume that the domain of the function is the set of consecutive integers starting with 1.

The first term is given, \( f(1) = 3 \). Use \( f(1) \) to find \( f(2) \), \( f(2) \) to find \( f(3) \), and so on. In general, \( f(n - 1) \) refers to the term that precedes \( f(n) \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( f(n - 1) + 2 )</th>
<th>( f(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

The first 4 terms are 3, 5, 7, and 9.
Questioning Strategies

- What is the fifth term of \( f(n) = n^2 + 1 \)? 26
- What sequence is generated by the rule \( f(n) = 3n \) when the domain is the set of consecutive integers starting with 1? Explain your reasoning. The sequence generated is 3, 6, 9, 12, ...; each term is three times the corresponding position number.
- What is the tenth term of the sequence \( f(n) = 3n \) when the domain is the set of consecutive integers starting with 1? Explain how to find it. 30; evaluate 3n for \( n = 10 \).

Avoid Common Errors

Be sure students pay attention to the domain of the function that defines a sequence. While 1 is typically used as the first number in the domain, some other number (typically 0) can be used instead. A change in the function’s inputs will result in a change in the function’s outputs, thereby creating a different sequence. For instance, if the domain of \( f(n) = n^2 + 1 \) is \{0, 1, 2, 3, ...\} rather than \{1, 2, 3, 4, ...\}, then the function generates the sequence 1, 2, 5, 10, ... rather than the sequence 2, 5, 10, 17, ... .

EXTRA EXAMPLE

Write the first four terms of the sequence \( f(n) = n^3 + 1 \). Assume that the domain of the function is the set of consecutive integers starting with 1. The first four terms are 2, 9, 28, and 65.

Questioning Strategies

- Is it possible to generate the second term of the sequence \( f(n) = f(n - 1) + 2 \) without knowing that \( f(1) = 3 \)? Explain. No; since the second term is calculated by using the term before it, it is necessary to know the first term.
- How is the sequence \( f(n) = 3n \) related to the sequence \( f(1) = 3 \) and \( f(n) = f(n - 1) + 3 \) for \( n \geq 2 \)? Both rules generate the same sequence: 3, 6, 9, 12, ....
- If you know only that the rule for the sequence is \( f(n) = f(n - 1) + 2 \) and that \( f(4) = 9 \), is it possible to determine that \( f(1) = 3 \)? Yes, if you know that \( f(4) = 9 \), the recursive rule tells you that adding 2 to \( f(3) \) gives \( f(4) \), so \( f(3) = 7 \). Again, the recursive rule tells you that adding 2 to \( f(2) \) gives \( f(3) \), so \( f(2) = 5 \). Finally, the recursive rule tells you that adding 2 to \( f(1) \) gives \( f(2) \), so \( f(1) = 3 \).

Differentiated Instruction

Visual learners may benefit by drawing diagrams that illustrate how the \( n \)th term of a sequence with a recursive rule is generated by the terms that come before it. Students can use arrows to show how previous terms become the inputs to generate the terms that follow.
Writing General Rules for Arithmetic Sequences

Use the arithmetic sequence 6, 9, 12, 15, 18, ... to help you write a recursive rule and an explicit rule for any arithmetic sequence. For the general rules, the values of \( n \) are consecutive integers starting with 1.

A. Find the common difference.

B. Write a recursive rule.

C. Write an explicit rule.

### EXPLORE

#### Writing Rules for an Arithmetic Sequence

The table shows end-of-month balances in a bank account that does not earn interest. Write a recursive and an explicit rule for the arithmetic sequence described by the table.

<table>
<thead>
<tr>
<th>Month</th>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Account Balance ($)</td>
<td>( f(n) )</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>140</td>
</tr>
</tbody>
</table>

A. Find the common difference by calculating the differences between consecutive terms:

\[
egin{align*}
80 - 60 &= 20 \\
100 - 80 &= 20 \\
120 - 100 &= 20 \\
140 - 120 &= 20
\end{align*}
\]

The common difference, \( d \), is \( 20 \).

B. Write a recursive rule for the sequence:

\[
f(1) = 60 \quad \text{and} \quad f(n) = f(n - 1) + 20 \quad \text{for} \quad n \geq 2
\]

C. Write an explicit rule for the sequence by writing each term as the sum of the first term and a multiple of the common difference.

\[
f(n) = 60 + 20(n - 1)
\]

### REFLECT

3a. Describe how to find the 12th term of the sequence.

Add 2 to the 11th term of the sequence.

3b. Suppose you want to find the 50th term of a sequence. Would you rather use a recursive rule or an explicit rule? Explain your reasoning.

Sample answer: explicit, with an explicit rule, you can calculate the 50th term directly; with a recursive rule, you have to find the first 49 terms before you can find the 50th term.

In an arithmetic sequence, the difference between consecutive terms is constant. The constant difference is called the common difference, often written as \( d \).

#### Writing General Rules for Arithmetic Sequences

Use the arithmetic sequence 6, 9, 12, 15, 18, ... to help you write a recursive rule and an explicit rule for any arithmetic sequence. For the general rules, the values of \( n \) are consecutive integers starting with 1.

A. Find the common difference.

B. Write a recursive rule.

C. Write an explicit rule.

### EXPLORE

#### Writing General Rules for Arithmetic Sequences

Use the arithmetic sequence 6, 9, 12, 15, 18, ... to help you write a recursive rule and an explicit rule for any arithmetic sequence. For the general rules, the values of \( n \) are consecutive integers starting with 1.

A. Find the common difference.

B. Write a recursive rule.

C. Write an explicit rule.
Questioning Strategies

• What would be the meaning in the context of the situation if $f(1)$ was equal to 20 instead of 60 in the recursive rule? It would mean that the starting balance was $20.

• In part C, why does the explicit rule for the sequence involve multiplying the common difference by $n - 1$ and not $n$? Month 1 ($n = 1$) represents the initial account balance, $60. A monthly deposit of $20 (the common difference) is made starting with month 2 ($n = 2$). So, the explicit rule must generate the sequence $f(1) = 60 + 0 \cdot 20, \ f(2) = 60 + 1 \cdot 20, \ f(3) = 60 + 2 \cdot 20, \ldots$ The number of 20s added to 60 is always 1 less than the month number.

• Which rule, the recursive rule or the explicit rule, would be more useful in determining the balance at the end of the 45th month? Explain your reasoning. The explicit rule is more useful because you can easily evaluate $f(45) = 940$ to find the balance at the end of the 45th month.

Differentiated Instruction
Kinesthetic learners may benefit by acting out the situation using counters to represent the money in the account.

EXTRA EXAMPLE

The table shows end-of-month balances in a bank account that does not earn interest. Write a recursive rule and an explicit rule for the arithmetic sequence described by the table.

<table>
<thead>
<tr>
<th>Month</th>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Account Balance ($)</td>
<td>$f(n)$</td>
<td>50</td>
<td>80</td>
<td>110</td>
<td>140</td>
<td>170</td>
</tr>
</tbody>
</table>

$f(1) = 50$ and $f(n) = f(n - 1) + 30$ for $n \geq 2$; $f(n) = 50 + 30(n - 1)$

Questioning Strategies

• What is a recursive rule for an arithmetic sequence with a first term of 10 and a common difference of $-4$? $f(1) = 10$ and $f(n) = f(n - 1) - 4$ for $n \geq 2$

• What is an explicit rule for an arithmetic sequence with a first term of $\frac{1}{2}$ and a common difference of $\frac{3}{4}$? $f(n) = \frac{1}{2} + \frac{3}{4}(n - 1)$

• If you know the second term and the common difference of an arithmetic sequence, can you write an explicit rule for the sequence? If so, explain how. Yes; you can subtract the common difference from the second term to get the first term. Then, you can substitute the first term and the common difference into the general explicit rule to get the explicit rule for the sequence.

Teaching Strategy

After students have written the general algebraic formulas for recursive and explicit rules for arithmetic sequences, review the process for substituting values for variables in a formula. In both the general recursive and explicit rules, values must be inserted for $f(1)$, the first term in the sequence, and $d$, the common difference. Remind students that the qualification $n \geq 2$ must be included in the recursive rule and guide students to understand why this is the case. Also, point out that the common difference $d$ is added to $f(n - 1)$ in the recursive rule, while $d$ is multiplied by $n - 1$ in the explicit rule.

Mathematical Practice

Highlighting the Standards

5 EXPLORE and its Reflect questions address Mathematical Practice Standard 2 (Reason abstractly and quantitatively). Students use a specific arithmetic sequence to help them obtain general rules, both recursive and explicit, for arithmetic sequences. They can then use the general rules to obtain rules for other specific arithmetic sequences and to transform a rule given in one form into the other form (recursive to explicit or explicit to recursive).
5a. The first term of an arithmetic sequence is 4 and the common difference is 10. Explain how you can find the 6th term of the sequence.

Evaluate the explicit rule for an arithmetic sequence when \( n = 6 \):

\[ f(n) = 4 + 10(n - 1) = 14 \]

5b. What information do you need to know in order to find the 4th term of an arithmetic sequence by using its recursive rule?

The 7th term and the common difference

5c. What is the recursive rule for the sequence \( f(n) = 2 + (-7)n - 1 \)?

\[ f(1) = 2 \text{ and } f(n) = f(n-1) - 3 \text{ for } n \geq 2 \]

### Example: Relating Arithmetic Sequences and Functions

The graph shows how the cost of a rafting trip depends on the number of passengers. Write an explicit rule for the sequence of costs.

4. Represent the sequence in a table.

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(n) )</td>
<td>75</td>
<td>100</td>
<td>125</td>
<td>150</td>
</tr>
</tbody>
</table>

3. Examine the sequence.

Is the sequence arithmetic? Explain how you know.

Yes; the difference between consecutive terms is constant.

What is the common difference?

25

C. Write an explicit rule for the sequence.

\[ f(n) = f(1) + (n - 1) \cdot d \]

Write the general rule.

\[ f(n) = 75 + 25(n - 1) \]

Substitute 75 for \( f(1) \) and 25 for \( d \).

So, the sequence has the rule \( f(n) = 75 + 25(n - 1) \) where \( n \) is the number of passengers and \( f(n) \) is the cost of the trip in dollars

### Reflect

6a. An arithmetic sequence is equivalent to a function with a restricted domain.

On the graph above, draw the line that passes through the given points. Then write a function of the form \( f(x) = mx + b \) for the line that you drew and give the function’s domain.

\[ f(x) = 25x + 50 \text{ for } x = 1, 2, 3, 4 \]

6b. Show that the explicit rule for the sequence is equivalent to the function.

Justify the steps you took.

\[ f(1) = 75 + 25(1 - 1) \quad \text{Explicit rule} \]
\[ f(2) = 75 + 25(2 - 1) \quad \text{Distributive property} \]
\[ f(3) = 75 + 25(3 - 1) \quad \text{Commutative and associative properties} \]

6c. A function of the form \( f(x) = mx + b \) is called a linear function because its graph is a line. Using the line you drew, what is the relationship between \( m \) and the common difference of the arithmetic sequence?

\( m \) is equal to the common difference.

### Practice

Write the first four terms of each sequence. Assume that the domain of the function is the set of consecutive integers starting with 1.

1. \( f(n) = (n - 1)^2 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(n) )</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

2. \( f(n) = \frac{n + 1}{n - 1} \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(n) )</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

3. \( f(n) = 40\sqrt{n} \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>0.5</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(n) )</td>
<td>40</td>
<td>20</td>
<td>8</td>
</tr>
</tbody>
</table>

4. \( f(n) = \sqrt{n - 1} \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(n) )</td>
<td>-,</td>
<td>1</td>
<td>0.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>

5. \( f(n) = 2 + f(n - 1) + 10 \) for \( n \geq 2 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(n) )</td>
<td>12</td>
<td>22</td>
<td>32</td>
<td>42</td>
</tr>
</tbody>
</table>

6. \( f(n) = 16 + f(n - 1) \) for \( n \geq 2 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(n) )</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
</tr>
</tbody>
</table>

7. \( f(n) = 1 + f(n - 1) + 1 \) for \( n \geq 2 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(n) )</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

8. \( f(n) = f(2) = 1 \) and \( f(n) = f(n - 1) + 3 \) for \( n \geq 3 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(n) )</td>
<td>0</td>
<td>-2</td>
<td>-4</td>
</tr>
</tbody>
</table>

Write the 12th term of each sequence. Assume that the domain of the function is the set of consecutive integers starting with 1.

9. \( f(n) = 3n - 2 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(n) )</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>

10. \( f(n) = 2n(n + 1) \)

    | \( n \) | 1 | 2 | 3 | 4 |
|-------|---|---|---|---|
| \( f(n) \) | 2 | 12 | 36 | 70 |
Questioning Strategies
• How is the domain of a function restricted if the function defines an arithmetic sequence?
The domain is restricted to a subset of the set of integers.

EXTRA EXAMPLE
If the points plotted on the graph in [EXAMPLE] were changed to (1, 25), (2, 75), (3, 125), and (4, 175), what explicit rule would you write for the sequence shown in the graph?

\[ f(n) = 25 + 50(n - 1) \]

Essential Question
Why is a sequence a function and how can you write a rule for an arithmetic sequence?
Each term in a sequence is associated with a position number, usually the set of consecutive integers starting with 1 or 0. Since each position number is associated with exactly one term, a numerical sequence is a function in which the domain is the set of position numbers and the range is the set of terms.

To write a recursive rule, assume \( f(1) \) is given and use the general formula \( f(n) = f(n - 1) + d \) for \( n \geq 2 \), where \( d \) is the common difference. To write an explicit rule, use the general formula \( f(n) = f(1) + d(n - 1) \), where \( f(1) \) is the first term in the sequence and \( d \) is the common difference.

Summarize
Have students make a graphic organizer outlining the process for writing explicit and recursive rules for sequences. Include details about how the first term of the sequence and the common difference are incorporated into each rule.

<table>
<thead>
<tr>
<th>Where skills are taught</th>
<th>Where skills are practiced</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 [EXAMPLE]</td>
<td>EXS. 1–4, 9, 10</td>
</tr>
<tr>
<td>3 [EXAMPLE]</td>
<td>EXS. 5–8</td>
</tr>
<tr>
<td>4 [EXAMPLE]</td>
<td>EXS. 17–20</td>
</tr>
<tr>
<td>6 [EXAMPLE]</td>
<td>EX. 21</td>
</tr>
</tbody>
</table>

**Exercises**

**Exercises 11–13:** Given a table that represents a function, students write an explicit rule for the sequence defined by the function.

**Exercises 14–16:** Given a table that represents a function, students write a recursive rule for the sequence defined by the function.

**Exercise 22:** Students write an explicit rule for an arithmetic sequence to solve a problem that involves finding the \( n \)th term of the sequence.
20. Carrie borrowed money to pay for a car repair. She is repaying the loan in equal monthly payments.

<table>
<thead>
<tr>
<th>Monthly Payment Number</th>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan Balance ($)</td>
<td>f(n)</td>
<td>840</td>
<td>720</td>
<td>600</td>
<td>480</td>
<td>360</td>
</tr>
</tbody>
</table>

a. Explain how you know that the sequence of loan balances is arithmetic.

b. Write recursive and explicit rules for the sequence of loan balances.

21. The graph shows the lengths of the rows formed by various numbers of grocery carts when they are nested together.

a. Write an explicit rule for the sequence of row lengths.

b. What is the length of a row of 25 nested carts?

22. Each stair on a staircase has a height of 7.5 inches.

a. Write an explicit rule for an arithmetic sequence that gives the height (in inches) of the nth stair above the base of the staircase.

b. What is the fourth term of the sequence, and what does it represent in this situation?

Notes

Write an explicit rule for each sequence. Assume that the domain of the function is the set of consecutive integers starting with 1.

<table>
<thead>
<tr>
<th>n</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

\( f(n) = n + 5 \)

Write a recursive rule for each sequence. Assume that the domain of the function is the set of consecutive integers starting with 1.

<table>
<thead>
<tr>
<th>n</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

\( f(n) = 3n \)

Write a recursive rule and an explicit rule for each arithmetic sequence.

17. 3, 7, 11, 15, ...

18. 10, 8, 6, 4, ...

19. 1, \( \frac{3}{2} \), \( \frac{5}{2} \), ...

23. Carrie borrowed money to pay for a car repair. She is repaying the loan in equal monthly payments.

<table>
<thead>
<tr>
<th>Monthly Payment Number</th>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan Balance ($)</td>
<td>f(n)</td>
<td>840</td>
<td>720</td>
<td>600</td>
<td>480</td>
<td>360</td>
</tr>
</tbody>
</table>

a. Explain how you know that the sequence of loan balances is arithmetic.

b. Write recursive and explicit rules for the sequence of loan balances.

\( f(n) = 840 \) and \( f(n) = f(n - 1) + 120 \) for \( n \geq 2 \); \( f(1) = 840 \) and \( f(n) = f(n - 1) + 120(n - 1) \) for \( n \geq 2 \)

c. How many months will it take Carrie to pay off the loan? Explain.


\$960; the initial balance is \( f(1) = 960 \).
Answers

Additional Practice

1. arithmetic; \(d = 3; 2, 5, 8\)

2. arithmetic; \(d = 1.5; 6, 7.5, 9\)

3. not arithmetic

4. arithmetic; \(d = -0.5; -22, -22.5, 23\)

5. \(-108\)  
6. \(23\)

7. \(97.8\)  
8. \(-60.8\)

9. \(-34.5\)  
10. \(73.8\)

11. \(\$213.40\)  
12. \(\$25.00\)

Problem Solving

1. \(\$1430\)  
2. \(1.95\) ounces

3. \(\$9400\)  
4. \(\$2.75\)

5. \(D\)  
6. \(J\)

7. \(A\)  
8. \(J\)

9. \(B\)
**Problem Solving**

Find the indicated term of each arithmetic sequence.

1. Darrell has a job and his savings have grown over the past several weeks.

<table>
<thead>
<tr>
<th>Weeks</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings</td>
<td>$130</td>
<td>$250</td>
<td>$360</td>
<td>$450</td>
</tr>
</tbody>
</table>

How much will Darrell have saved after 11 weeks?

3. A new car costs $15,000 and is depreciating by $900 each year. How much will the car be worth after 4 years?

5. Which of the following shows how many ants Ivan will have in the next three weeks?

A. 315, 341, 367  
B. 317, 343, 369  
C. 318, 334, 350  
D. 319, 345, 371

6. Which rule can be used to find how large the colony will be in n weeks?

A. $a_n = 215 - 2n$  
B. $a_n = 215 + 26n$  
C. $a_n = 215(n - 1) + 26$  
D. $a_n = 215 - 26(n - 1)$

7. How many ants will Ivan have in 27 weeks?

A. 891  
B. 917  
C. 5616  
D. 5631

8. Ivan’s ants weigh 1.5 grams each. How many grams do all of his ants weigh in 13 weeks?

A. 860.5  
B. 983  
C. 790.5  
D. 722

9. When the colony reaches 1385 ants, Ivan’s ant farm will not be big enough for all of them. In how many weeks will the ant population be too large?

A. 45  
B. 48  
C. 47  
D. 46